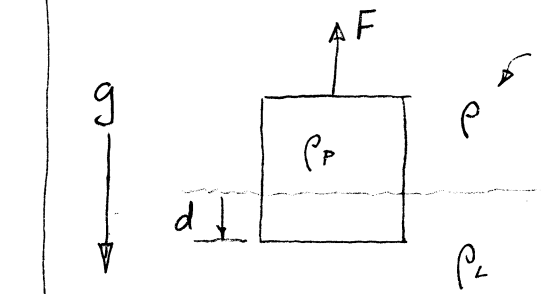
UE Fluids Quiz 1 Solution Fall '07

25 **1a)**

Parameters		Units
force:	F	$kg\cdot m/s^2$
liquid:	ρ_L	kg/m^3
plastic:	ρ_p	kg/m^3
air:	ρ	kg/m^3
gravity:	g	m/s^2
length	ℓ	m
depth	d	m
$N=7$		$k=3$



Air buoyancy is probably very small, but measurable

25 pts

25 **1b)** $N-k = 7-3 = 4$ Π groups

$$\Pi_1 = \frac{F}{\rho_p \ell^3 g}, \quad \Pi_2 = \frac{\rho_L}{\rho_p}, \quad \Pi_3 = \frac{\rho}{\rho_p}, \quad \Pi_4 = \frac{d}{\ell}$$

Many alternative sets are possible, as usual.

25 pts

20 **2a)** $$C_\ell \equiv \frac{L'}{q_\infty c} \simeq \frac{1}{q_\infty c}\int_0^c (p_\ell - p_u)\,dx = \int_0^1 \left[\frac{p_\ell - p_\infty}{q_\infty} - \frac{p_u - p_\infty}{q_\infty}\right] d\left(\frac{x}{c}\right) = \int_0^1 (C_{p_\ell} - C_{p_u})\, d\left(\frac{x}{c}\right)$$

$$C_\ell = \int_0^1 \left[0.5\left(1-\frac{x}{c}\right) + 1.0\left(1-\frac{x}{c}\right)\right] d\left(\frac{x}{c}\right) = 1.5\int_0^1 \left(1-\frac{x}{c}\right) d\left(\frac{x}{c}\right) = 1.5\left[\frac{x}{c} - \frac{1}{2}\left(\frac{x}{c}\right)^2\right]_0^1 = 0.75$$

This made the assumption that $L' = N'\cos\alpha - D'\sin\alpha \simeq N'$ since $\cos\alpha \simeq 1$, $\sin\alpha \simeq 0$

20 pts

20 **2b)** $$C_{m_{LE}} = \frac{M'_{LE}}{q_\infty c^2} = \frac{1}{q_\infty c^2}\int_0^c -(p_\ell - p_u)\,x\,dx = \int_0^1 -(C_{p_\ell} - C_{p_u})\left(\frac{x}{c}\right) d\left(\frac{x}{c}\right)$$; similar to above

$$C_{M_{LE}} = \int_0^1 -1.5\left(1-\frac{x}{c}\right)\frac{x}{c}\, d\left(\frac{x}{c}\right) = -1.5\int_0^1 \left[\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] d\left(\frac{x}{c}\right) = -1.5\left[\frac{1}{2}\left(\frac{x}{c}\right)^2 - \frac{1}{3}\left(\frac{x}{c}\right)^3\right]_0^1 = -0.25$$

Using moment translation relation: $\left[M'_{c/4} = M'_{LE} + \frac{c}{4}\cdot L'\right]\frac{1}{q_\infty c^2} \rightarrow C_{m_{c/4}} = C_{m_{LE}} + \frac{1}{4}C_\ell$

$$C_{m_{c/4}} = -0.25 + \frac{1}{4}0.75 = -0.0625 = -\frac{1}{16}$$

20 pts

10 **2c)** At x_{cp}: $M'_{cp} = M'_{LE} + x_{cp}\cdot L' = 0$ or $C_{m_{cp}} = C_{m_{LE}} + \frac{x_{cp}}{c}C_\ell = 0$

$$\frac{x_{cp}}{c} = -\frac{C_{m_{LE}}}{C_\ell} = -\frac{-0.25}{0.75} = \frac{1}{3}, \quad x_{cp} = \frac{c}{3}$$

10 pts